Student Name ___________________________________ ***Date Submitted*** _______________________

PRINCIPLES OF MATH 12 (v5)

Section 1.0 Send-In: *Graphing Calculator Review*

Complete this send-in as part of your course enrollment. This will be your first mark entered for the course. When this assignment has been received by SCIDES, your course materials will be sent to you.

This send-in consists of:

- Principles of Math 12 Course Planner ______/ 5 marks
- Guided Practice 1.1 ______/15 marks
- Guided Practice 1.2 ______/25 marks

TOTAL: ______/ 45 marks ______%

Mail:

1) This **Cover Sheet**
2) **Return Address** (page 2 or Comment Sheet) – Fill out with your complete name and address.
3) **Send-In Assignments** – Completed above noted assignments.

Be sure to put proper ***postage*** *on the envelope (if necessary) and add your* ***return address****.*

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You are REQUIRED to fill out your return address on this page if you DOWNLOADED this Assignment. [If the Assignment was mailed to you, please fill out the back of the white page of the 3 part District Education Comment Sheet.]

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Complete your MAILING address.

Is this a change of address? ☐ **Yes** ☐ **No**

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Name: ______________________________

___ / 5 marks

COURSE NAME Course Planner

Complete <u>all</u> the following contact information that applies to you and check the one that is the best way to contact you during the day:

❑ Home Phone: ______________________ ❑ Work Phone: ______________________ ❑ Cell: ______________________

❑ Email: ____

❑ other way to contact you (explain) ____

When is the best time for your teacher or tutor/marker to contact you? ___:____ AM PM

Check your Grade: ❑ Grade 10 ❑ Grade 11 ❑ Grade 12 ❑ Graduated

Timetable Options/Course Plan

One of the keys to being successful in anything that you do is to take the time to plan carefully. The objective of this section is to help you create a timetable for managing your schoolwork and enable you to set goals for finishing all of your courses by your desired completion date. **Most full-time students complete 3 to 5 assignments each week.**

The flexibility of our distributed learning program offers you many choices but a plan for completion is essential to success. Most full-time students complete 8 courses in a school year (10 months). The most common timetables are "semestered" (4 courses at a time) or "linear" (8 courses at a time).

What is your planned schedule? ❑ Semester System (22 weeks) ❑ Linear System (44 weeks)

❑ other: *(explain)* ____

What is your intended **start** date for this course? ❑ Now ❑ Other date: ________________________

What is your intended **completion** date for this course? _____ (month) _____ (year)

How many courses are you taking with us this year? ____ How many with other schools/programs? _____

COURSE NAME consists of XX more send-in assignments and XX tests. How many assignments/tests per week must you do to complete this course as planned? _______

- *Mark target submission dates on a calendar.*
- *Add this same information from other courses to help you create a schedule for completion.*
- *Record the actual dates you submit work so you can track your progress.*

Delivery Method

COURSE NAME is offered as a print course only. You will receive workbooks in print form and will be submitting your assignments through the regular mail.

If you have access to the Internet, you will find some great online resources to support your learning by searching for key words in the assignments.

Anything else?

Is there anything else you would like us to know about you or your education plans that will help us provide you with better service?

Lesson 1

GRAPHING CALCULATOR REVIEW

Outcomes

Upon completing this lesson, you will be able to carry out these operations on a graphing calculator:

- Enter and edit polynomial equations
- Graph the equations and adjust the viewing window
- Solve the equations

Overview

Graphing calculators became part of Principles of Math 12 in 1999, and then part of Principles of Math 11 in September 2001. Students who completed their Principles of Math 11 prior to that date likely did not use a graphing calculator. This review lesson gives such students a quick introduction to using one as they begin Principles of Mathematics 12.

If you used a graphing calculator for Principles of Math 11, then consider this lesson optional. You should do just a few of the exercises to make sure you remember how the graphing and solving functions work.

The directions here are specific to the **TI-83** or **TI-83Plus** models from Texas Instruments. You may be using a different TI model, or one made by Hewlett-Packard, Sharp, Casio, or another manufacturer. Any graphing calculator will get you through the provincial exam (except the HP-48, which is not allowed). But if you use another brand of calculator, you will need to refer to its user manual to find out how to do what these instructions tell you.

Q: Can I use an ordinary *scientific* calculator for the exams in this course and for the Provincial exam?

A: Not much. You are allowed to bring one to exams if you want to, but it's *not enough*. The calculator you bring to the Provincial exam must be able to *display graphs* in its display window, as well as solve (calculate and display the roots of) equations.

You can distinguish a graphing calculator by the size of its display window. All graphing calculators have a large (for a calculator) rectangular display: about 4 cm high by 6 cm wide. If your calculator's display is less than 3 cm high, it won't work for the provincial exam.

The graphing calculator also includes all the functions of a scientific calculator. While you may bring two calculators to exams (one scientific, one graphing), most students find that the graphing calculator is the only one they need.

Q: Do I still need to be able to solve equations by algebra, i.e., by other methods I have been taught that do not use a calculator?

A: Yes! The provincial exam always includes questions which take a long time to solve on a calculator, but which you can solve rapidly by hand if you know the rules. Do not plan on using your calculator whenever equations show up on the exam—that will take too long. As in real life, the calculator is for problems which *cannot* be easily solved by hand.

Solving Polynomial Equations Using the Graphing Calculator

You have developed considerable skill at finding the rational (which includes integral) roots of given polynomial equations. But the roots of a polynomial equation are *not* necessarily rational. They might well be irrational numbers (non-periodic non-terminating decimals). This possibility can make the algebraic solution of the equation very tedious. The graphing calculator can simplify the process.

A word of caution as we begin: enter values and follow the steps *slowly* and *carefully*. The calculator has no tolerance for entry errors, no matter how small.

Also, negative numbers must be signed using the negation [(-)] button just to the left of the [ENTER] button, not the subtract [–] operation button; otherwise you'll get a "syntax error" message. (On other brands of calculators, the [(-)] key may be a [±] or [±/–] key.)

In this lesson, we use two sizes of hyphens to distinguish between the negation key and the subtract or minus key, just as the TI calculators do. For negation we use a short hyphen [-] and for subtraction we use a longer one [–]. (After this lesson, we'll use just the longer dash in all equations; you will know the rule by then for choosing the correct key.)

> **Q:** What do I do when the calculator says "Syntax error"?
>
> **A:** Choose option 2 "Goto" from your screen. The blinking cursor will go directly to the error you made so that you can fix it.

As you go through the following examples, perform the steps on your calculator rather than just reading the text. You might want to go over the example a number of times until you feel comfortable with the functions. As with any skill, practice makes perfect.

If you make typing errors at any time, you can always scroll to your error using the four cursor (arrowhead) keys and then:

1) type over,
2) use the [DEL] key to delete, or
3) use the Insert function (by pressing [2nd] and then [DEL]), and then type more characters in the same space.

Note: Upon first turning on your graphing calculator, you should see a blank display—if you don't, press CLEAR. In this mode, your graphing calculator functions as any scientific calculator does, thus enabling you to solve such equations as 2 + 2 or sin 25.

Example 1

Solve the equation $3x^3 - 13x^2 - 10x = -50$

Solution

First we rearrange the equation (on paper) so that we have zero on one side of the equation:

$3x^3 - 13x^2 - 10x + 50 = 0$

Turn on your calculator and ensure that the memory is cleared by pressing 2nd, then +, then scroll down to option "**3:Clear Entries**" using the down arrowhead key and select that option by pressing ENTER. Now you will see a confirmation screen, so you press ENTER while the cursor is next to the words **Clear entries**. You will see the word **Done**. Press CLEAR to get a blank screen.

Shortcuts

You can select the menu options simply by pressing their number, if you prefer not to scroll through the other options.

The 2nd key is used in the same way as it is used on a regular calculator in that it performs the function shown ***above*** the keys.

The X,T,θ,n Key

The X,T,θ,n key is the one you use to insert a *variable* into the equation you type. To type "sin θ", you hit the sin key, then the X,T,θ,n key. Then close the parenthesis. This key also inserts the "x" into polynomial equations.

{Braces}

To type a brace, use the 2nd key and the corresponding parenthesis. Most users don't bother with braces since "nested" parentheses work just as well. Equations 1 and 2 mean the same thing on a graphing calculator:

1. $\left[3\left(1-\frac{x^2}{3}\right)\right]$

2. $\left(3\left(1-\frac{x^2}{3}\right)\right)$

Equation 1 is easier for humans to read, but equation 2 is easier to enter on the calculator—fewer keys to press.

Step 1: To solve for the roots of this equation, we will solve for the zeros of the corresponding polynomial function $Y_1 = 3x^3 - 13x^2 - 10x + 50$. We begin by typing in the function as follows:

Press [Y=]. This bring up a flashing cursor to the right of **Y₁=** in your display window.

Step 2: At the flashing cursor we begin typing in our function carefully:

3 [X,T,θ,n] [^] 3 [–] 13 [X,T,θ,n] [^] 2 [–] 10 [X,T,θ,n] [+] 50

Although spaces have been used between the above numbers for clarity, don't type spaces on the graphing calculator. Here's how your display should look:

Y_1 =3X^3–13X^2–10

X+50

Y_2=

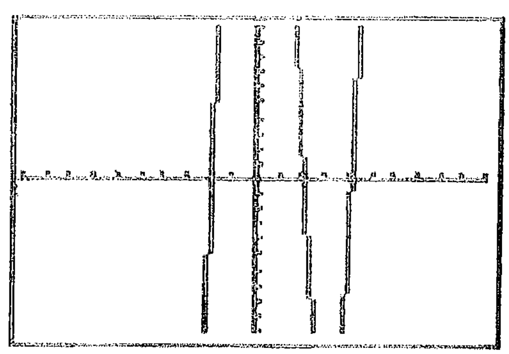
Y_3=

Y_4=

Our font is a little different from that of the graphing calculator but hopefully you get the picture. Notice the use of the [^] key. It indicates to the graphing calculator that the operation is a power/exponent.

Step 3: Now we graph the polynomial function defined in Step 2 by pressing [GRAPH].

You should see a display similar to the one shown here:



Graph 1: $3x^3 - 13x^2 - 10x = -50$

Step 4: We identify the approximate value(s) of the zero(s) by inspection of the graph. Sometimes we need to adjust the viewing window a little so that we can see where/if the x-intercepts or zeros occur, but in this case all ***three*** are visible. A cubic polynomial can have, at most, three Real zeros, so we need not worry that some are not visible.

a) The least zero is in the interval {−3,−1} (i.e., between −3 and −1). A guess might be −2.0.

b) The middle zero is in the interval {2,3} (between 2 and 3). A guess might be 2.5.

c) The greatest zero is in the interval {4,5} (between 4 and 5). A guess might be 4.5.

Step 5: Now we will solve for the actual zeros, one by one. The calculator needs a few details. The TI-83 will expect to receive them in this very specific order[†]:

Function, Variable, Guess, {Lower bound, Upper bound}

At this point, different calculators use different key sequences to solve equations. The remaining steps 6–10 are for the TI-83. If you are using a different calculator, look in the index of its user manual under "Solving equations".

Be sure to:

- use the [X,T,θ,n] key for X.
- use the minus key within the equation itself.
- use the negation [(-)] key for the guess and the bounds.

[†] Want to know the math behind the button? Calculators and computers solve functions with some form of Isaac Newton's method. On a graph, Newton's method finds solutions or zeros (x-axis crossing points) like this: using your initial "guess" and the slope of the graph at the point of your guess, it calculates where that slope (a straight line, of course) crosses the x-axis. Then it takes the crossing point as a second guess—it goes to the graph point directly 'above' or 'below' where the first slope crossed the x-axis. It calculates the new slope of the equation at that point, goes to where that new slope crosses the x-axis, and repeats the process.

If a guess is close to a zero or solution—to a point where the graph crosses the x-axis—you can see that the graph's slope from that point will be almost parallel to the graph itself; the slope's crossing point on the x-axis will be close to the crossing point of the graph. Only a few repeats will be needed, before it homes in on the actual crossing point. In reality, graphing calculators perform so many repeats of Newton's method in the time it takes to press one button that your first guess need not be all that close to a true solution. Just make sure that your first guess is clearly closer to *one* solution (or crossing point) than it is to any other solutions. Better yet, set the Lower and Upper Bound so that they contain only one crossing point. If your guess is sort of midway between two crossing points, you can't control which one Newton's method will find!

Do type in the commas, and ***do not*** use spaces.

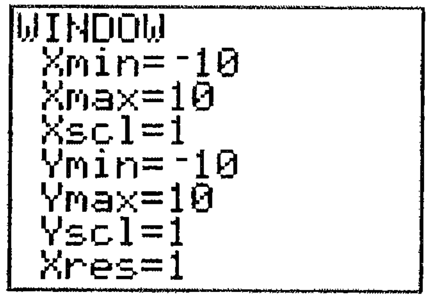
	TI-83
Step 6	Press [MATH], scroll down to option 0:Solver.... Select it with [ENTER].
	You should now see: **EQUATION SOLVER** **Eqn:0=** If you do not see EQUATION SOLVER, scroll up. If you see an equation already written in, use the cursor-up (up-arrow) key to place the cursor on the equation. Then use the [CLEAR] key to remove it.
Step 7	Type the following equation exactly. **3X^3 – 13X^2 – 10X + 50** Then press [ENTER] Next to the **X=** on the next line, type your first guess: –2.0 (remember to use the negation key, not minus). On the **bound=** line, type your lower and upper bound (for the first guess) between *braces*, like this: {–3, –1} Finally, place your cursor on the **X=** line and press [ALPHA] [SOLVE]. (ALPHA is the green key near [2nd], and SOLVE is on the [ENTER] key)
Step 8	Your answer appears: **X=–1.920589771** which we round to –1.921. If your answer is different, look carefully at your equation for mistakes. Use the cursor keys to locate and correct them.
Step 9	Go back to Step 7. Change your **bound=** line to {2,3} and your X= guess to 2.5. Then press [ALPHA] [SOLVE]. Ensure you get the answer **2.07815274** which rounds to 2.078. If you got a **BAD GUESS** message, it means your guess is outside your bounds.
Step 10	Go back to Step 7 and use your third guess of 4.5 and bounds of 4 and 5. Then press [ALPHA] [SOLVE]. Ensure you get the answer **4.175770562** which rounds to 4.176.

So the three solutions for the equation $3x^3 - 13x^2 - 10x = -50$ from least to greatest are: –1.921, 2.078, and 4.176.

Adjusting the Viewing Window

Finally, we adjust the "viewing window" on the calculator so that more of the graph is visible. Recall that the graph you saw on your calculator (see page 9) went off the top and bottom of the screen—you could not see it all.

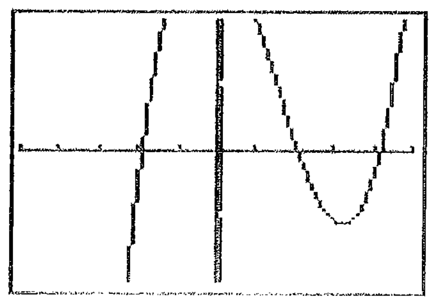
To fix that, you adjust the Viewing Window. Press the WINDOW button at the top of the keyboard. You see the list of values at the right. From the top, these values tell you that the X-axis in your window ranges from –10 to +10, with a tic-mark for every number. The same is true for the Y-axis.



Co-ordinate values for the Standard Viewing Window on the TI-83.

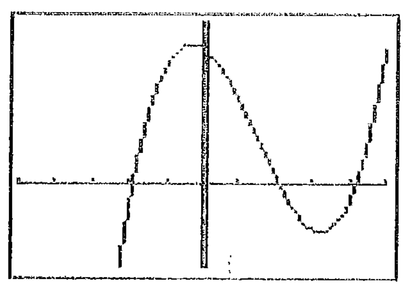
For your graph to "look right," adjust the scale so that there are fewer values displayed along the X-axis (from –5 to +5, say) and more displayed along the Y-axis.

As a first guess, use your cursor to set Xmin = –5 and Xmax = 5. Likewise, set Ymin = –30 and Ymax = 30. Then press GRAPH again. Now the graph looks like the one at right.



The graph with the WINDOW parameters at Ymin = –30 and Ymax = 30

This is better, but we're still missing the upper loop of the graph. Our final adjustment is to change Ymax to +60. That yields the more appropriate display to the lower right. We could change the Xmin value from –5 to –3 for a more balanced look, but what we have is good enough.



X [–5, 5] Y [–30, 60]

When you answer graphing-calculator questions on the Provincial Exam, you will hand-sketch the graph in your calculator's viewing window, then write in the Xmin and Xmax, and Ymin and Ymax values, which you set for your window. The example at right shows the correct way to write your window settings.

Guided Practice 15 marks

Solve each of the equations below using your graphing calculator and showing the following detail:

Sketch the display as you see it.

For *each* of the possible solutions:

a) state your "guess".

b) indicate the upper and lower bounds for x using interval notation. For example (5,7) would indicate that x falls between 5 and 7.

c) state the actual solutions, correct to three decimal places.

Your graph sketch goes here.

1. $x^3 - x^2 - 12x = -3$

X [,] Y[,]

2. $-x^3 + 2x^2 - x + 1 = 0$

X [,] Y[,]

3. $x^3 + 6x^2 + 3x - 5 = 0$

X [,] Y[,]

4. $x^3 - 3x^2 = 9x - 9$

X [,] Y[,]

5. $0.25x^3 - 0.5x^2 - 6x - 2 = 0$

X [,] Y[,]

Lesson 2

Functions and Interval Notation

Outcomes

Upon completing this lesson, you will be able to:

- identify the domain and range of various functions using set notation and interval notation
- find the x- and y-intercepts of any function
- perform operations on functions

Overview

The concepts of domain and range are necessary to describe functions. Interval notation is a most convenient way to describe domain and range. We will also review combinations of functions and composition of functions.

Definitions

A ***function*** is a relation where each x-value has only one y-value

For any function, $y = f(x)$, the ***domain*** is the set of possible x-values, and the ***range*** is the set of possible y-values.

We can evaluate a function at a particular point by substituting either numbers or algebraic constants into the $f(x)$ expression and simplifying the result.

Example 1

If $f(x) = x^3 - 2x$, find $f(0)$, $f(-2)$, and $f(a)$

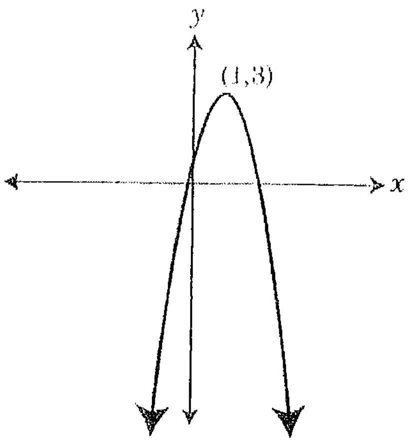
$f(0) = 0^3 - 2(0) = 0$

$f(-2) = (-2)^3 - 2(-2) = -8 + 4 = -4$

$f(a) = a^3 - 2a$

Example 2

State the domain and range of the function $p(x) = -2(x-1)^2 + 3$



There is no restriction on the values that x can take.

Domain = $\Re$

The parabola has a maximum value at (1,3)

Range = $\{y \mid y \leq 3, y \in \Re\}$

Turn to Appendix 1 for some background information on how to read set notation.

Informal Rules for Finding a Function's Domain and Range

There is no "sure-fire formula" to determine the extent of a function along the X-axis (its domain) or the Y-axis (its range). The domain and range are often obvious if you just look at the graph; here are some guidelines for finding the domain and range of a function by looking at the equation, not the graph.

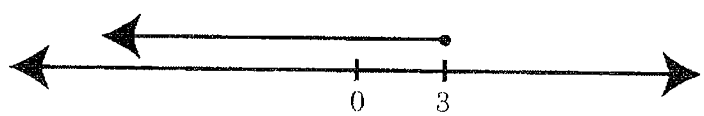
1. Linear functions, $f(x) = mx + b$, always have infinite domain and range, in both directions (i.e., out to $-\infty$ and also to $+\infty$). The only exception is when $m = 0$; then $y = b$ and the domain remains infinite but the range is simply b.
2. Parabolic functions, $f(x) = a(x-h)2 + b$, have infinite domain but a limited range—the range goes to infinity in one direction on the Y-axis, but not in the other direction. (This rule applies to any even-powered function: $x2$, $x4$, $x6$ and so on.) If the exact range boundary is not obvious from the equation, your graphing calculator can identify it for you; this is taught in Lesson 4.
3. In square-root functions (with x somewhere under the root sign), both the domain and range are infinite in one direction but not in the other.
4. In any function where x appears in the denominator of a fraction, watch out for specific values of x where the denominator becomes zero. At those points the equation has no meaning (and the graph shoots off to infinity along an

asymptote). That x-value must be excluded from the domain, and the corresponding $f(x)$, or y value (if there is one), must also be excluded from the range.

You may find these guidelines helpful as you begin Math 12 and work through the course. Most students find that domains and ranges become obvious enough that they don't need these guidelines for very long.

Example 3—Interval Notation

One way of reading the set $\{y \mid y \leq 3, y \in \Re\}$ is "All the real numbers between $-\infty$ and 3." On a number line, it would look like this:



We can write this interval from $-\infty$ up to and including 3 as $(-\infty, 3]$. The "(" means that the set *doesn't include* $-\infty$ (because infinity is unreachable) and the "]" means that the point 3 is *included* in the set.

So we see that another way of writing Range = $\{y \mid y \leq 3, y \in \Re\}$ is Range = $(-\infty, 3]$.

In this way we can rewrite $\Re$ as $(-\infty, \infty)$.

$\{x \mid -4 < x \leq 3\}$ as $(-4,3]$

$\{x \mid 10 \leq x \leq 12\}$ as $[10,12]$

$\{x \mid x \geq -2\}$ as $[-2,\infty]$

An interval where the end points are both included is called a **closed interval** and shown as [ ].

An interval where both end points are not included is called an **open interval** and shown as ( ).

An interval where only one end point is included is called a **half-open interval** and shown as either [ ) or ( ].

We can form the union of two intervals in the same way that we form the union of two sets. Remember that $\cup$ is the symbol for union.

$\{x \mid x < -5, x \in \Re\} \cup \{x \mid 0 < x < 4, x \in \Re\}$ can be written as $(-\infty,-5) \cup (0,4)$.

Most rational functions have restrictions because the denominator of a function cannot be zero. The domain and range both have restrictions. Interval notation is a convenient way to express a restricted range or domain.

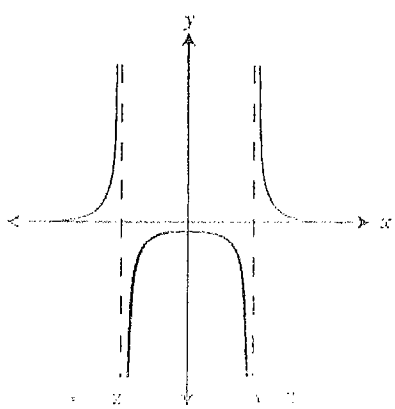
Example 4

The rational function $f(x)=\dfrac{1}{(x^2-4)}$ has asymptotes at $x = \pm 2$, and $y = 0$. It has a y-intercept at $y = -1/4$, but no x-intercepts.

Note: Remember, an asymptote is a line that a curve approaches to infinity but never never touches.

Domain = $\{x \mid x \neq -2, 2\}$. In interval notation this would be $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$, which is very awkward.

Range = $\{y \mid y \leq -1/4 \text{ or } y > 0\}$. In interval notation, this would be $(-\infty,-1/4)\cup(0,\infty)$.



Combination of Functions

Two functions can be combined arithmetically by +, −, × or ÷. The normal rules about addition, subtraction, etc. apply.

For example, if $f(x)=\dfrac{1}{2x-1}$ and $g(x)=\sqrt{3x-2}$, then we can create new combined functions by simple arithmetic like this:

$$f(x)+g(x)=(f+g)(x)=\frac{1}{2x-1}+\sqrt{3x-2}$$

Note: There are two ways to show the addition of functions: $f(x)+g(x)$ or $(f+g)(x)$.

Similarly, $(f-g)(x)=\frac{1}{2x-1}-\sqrt{3x-2}$

$$(f\times g)(x)=\frac{1}{2x-1}\times\sqrt{3x-2}=\frac{\sqrt{3x-2}}{2x-1}$$

$$(f\div g)(x)=\frac{1}{2x-1}\div\sqrt{3x-2}=\frac{1}{(2x-1)\sqrt{3x-2}}$$

Rule: When functions are combined arithmetically like that, the domain of the result is the *intersection* of domains from the two functions—it's the set of all points that belong in *both* the original domains. The same is true of the combined range—it's the *intersection* of the two separate ranges.

That intersection-of-domains rule applies for all four arithmetic operations, between any two functions. But the division operation (f÷g) has an additional rule: the combined domain and range cannot include any value that makes the new denominator go to zero.

Important definition!!

Whenever this course uses a plain square root sign, it refers only to the positive square root. This definition is common in most modern mathematics.

Examples: If you solve $x=\sqrt{4}$, then the answer is $x=2$ but *not* $x=-2$. If it wants the negative root, this course will ask you to solve $x=-\sqrt{4}$. If this course asks you for both roots, it will ask you to solve $x=\pm\sqrt{4}$.

Example 5

Using the above two functions *f* and *g*, find the domain and range of *f*+*g*, *f*–*g*, *f*×*g*, and *f*÷*g*.

Solution

By inspection, domain of $f=\{x\mid x\neq\frac{1}{2}\}$ and the range of $f=\{y\mid y\neq 0\}$. $x=\frac{1}{2}$ and $y=0$ are not allowed because *f* cannot have a zero in the denominator.

Similarly, domain of $g=[\frac{2}{3},\infty)$

Range of $g=[0,\infty)$

The square root of a negative number is not real so *g* cannot be less than $\frac{2}{3}$. But the value $\frac{2}{3}$ is in the domain of *g*.

Now for the intersections. Remember that ∩ is the symbol for intersection:

Domain of $(f+g)$ = domain of $(f-g)$ = domain of $(f\times g)$ = $\{x\mid x\neq\frac{1}{2}\}\cap[\frac{2}{3},\infty)=[\frac{2}{3},\infty)$.

Range of $(f+g)$ = range of $(f-g)$ = range of $(f\times g)$ = $\{y \mid y \neq 0\} \cup [0,\infty) = [0,\infty)$.

The domain of $(f\div g) = (\frac{2}{3},\infty)$ instead of $[\frac{2}{3},\infty)$. That's because $g(x)$ is in the denominator and $g(\frac{2}{3}) = 0$. So $\frac{2}{3}$ must be deleted from the combined domain.

The range of $(f\div g)$ is $(0,\infty)$, just as it is for $f+g$, $f-g$, and $(f\times g)$. The value 0 was excluded from the range of f already, so it's not going to appear in the range of the combined function.

Sometimes, when you write the range or domain of a combined function, it may be simpler to leave the intersection symbol $\cap$ in your answer.

Finding Intercepts

For more complex functions, we often need to know the x- and y-intercepts in order to find the domain and range.

Rule: To find the y-intercept, set $x = 0$. To find the x-intercept, set $y = 0$.

Example 6

Find the x- and y-intercepts for the function, $f(x) = x^2(x + 1)(x - 2)$

Solution

y-intercept: Set $x = 0$

$f(0) = 0^2(0 + 1)(0 - 2) = 0$

x-intercepts: Set $y = 0$

Solve: $x^2(x + 1)(x - 2) = 0$

$x^2 = 0$, or $x + 1 = 0$, or $x - 2 = 0$

$\therefore x = 0, -1, \text{ or } 2$

Composition of Functions

A composition of two functions is when they are arranged so that one is a function of the other.

Composition is not the same as "combination" using arithmetic operations between functions!

The composition is written as $(f \circ g)(x) = f(g(x))$ or as $(g \circ f)(x) = g(f(x))$, either with a small hollow circle for the operation, or with one function nested inside the other. In this course, $f(g(x))$ is the usual notation but we'll start by using both forms.

Example 7

If $f(x) = x - 3$ and $g(x) = 2x + 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x)$	$= f(g(x))$	
	$= f(2x + 1)$	Substitute formula for $g(x)$
	$= (2x + 1) - 3$	Apply formula for $f(x)$
	$= 2x - 2$	Simplify
$(g \circ f)(x)$	$= g(f(x))$	
	$= g(x - 3)$	Substitute formula for $f(x)$
	$= 2(x - 3) + 1$	Apply formula for $g(x)$
	$= 2x - 5$	Simplify

Notes to remember:

1. As Example 7 suggests, $(g \circ f)(x) \neq (f \circ g)(x)$ except in special cases.
2. For $(f \circ g)(x) = f(g(x))$, the range of g becomes the domain of f.

Example 8

If $f(x) = \frac{3}{2}x$ and $h(x) = x + 1$, write an equation for $(f \circ h)(x)$. Specify the domain and range.

Solution

$$(f \circ h)(x) = f(h(x))$$

$$= \frac{3}{(2x+1)}$$

To find the restrictions on the domain, remember that the denominator cannot be zero.

$$2(x+1) \neq 0$$

$$x \neq -1$$

To find the restrictions in the range, write the function as

$$y = \frac{3}{2(x+1)}, \text{ rearrange and solve for } y.$$

$$2y(x+1) = 3$$
$$2xy + 2y = 3$$
$$2xy = 3 - 2y$$
$$x = \frac{(3-2y)}{2y}$$

Restriction: $y \neq 0$

Domain of $(f \circ h) = \{x \mid x \neq -1\}$

Range of $(f \circ h) = \{y \mid y \neq 0\}$

Guided Practice [25 marks]

1. Given that $f(x) = 4x^2 - x + 3$ and $g(x) = 1 - 2x$, find: [11 marks]

   a) $f(0)$
   b) $g(0)$
   c) $f(-2)$
   d) $g(1/4)$
   e) $f(a)$
   f) $g(b - 1)$
   g) $(f+g)(x)$
   h) $(g - f)(x)$
   i) $(f \times g)(-2)$
   j) $(f \div g)(0)$
   k) $(g \div f)(b - 1)$

2. Determine the x- and y-intercepts for the following functions: [3 marks]

   a) $f(x) = 2x^2 - 8$
   b) $g(x) = \sqrt{2x + 5}$
   c) $k(x) = 5 - x$

3. Using the information from your answers to question 2. write the domain and range for each function using: [6 marks]

   i) set notation
   ii) interval notation

4. Given that $p(x) = \sqrt{x - 4}$ and $q(x) = 3x + 1$: [5 marks]

   a) determine each of the following:
      i) $p(q(x))$
      ii) $p(q(3))$
      iii) $q(p(a))$

   b) find the domain and range of:
      i) $p(q(x))$
      ii) $q(p(x))$